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Linear Quadratic Controller Design for the Deep Space Network Antennas

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A linear quadratic controller design procedure is proposed for NASA's deep space network antennas. The procedure is developed for an antenna model divided into tracking and flexible subsystems. Controllers for the flexible and tracking parts are designed by separately adjusting the performance index weights. First the weights for the flexible part are adjusted, next the weights for the tracking part are determined, followed by the flexible controller final tune-up. The controller for the flexible part is designed separately for each component; thus the design consists of weight adjustment for small-sized subsystems. The effect of weights on system performance is analytically described in this paper. The procedure is illustrated with the control system design for the recently built DSS-13 34-m dish antenna.

Introduction

A LINEAR quadratic (LQ) controller design procedure for the deep space network antennas is presented. Alvarez and Nickerson[1] have designed the LQ controller for the DSS-14 antenna. In their approach the gearbox flexible mode was included in the rigid-body model of the antenna. In recently designed antenna structures (such as the DSS-13 antenna in Fig. 1), significant flexible deformations are observed during tracking operations. The antenna rate-loop model described in Refs. 2 and 3 consists of 21 flexible modes up to 10 Hz. Controllers for these antennas should suppress flexible motion while following the tracking command. The LQ controller for the DSS-13 antenna was analyzed by Brekke.[4] The closed-loop system properties are achieved by iterative selection of design weights by defining the target zero locations and observing the command bandwidth. The method presented in this paper allows the design of a controller with a flexible motion suppression capability through sequential adjustment of the weights of the LQ performance index.

In Refs. 1 and 5, the controller is designed separately for the elevation and azimuth drives. This approach, effective for slow and/or rigid antennas, cannot be justified for fast and/or flexible antennas. In the latter case, the flexible properties of the full antenna significantly differ from the properties of the elevation-only or azimuth-only model of the antenna; thus the separate design of controllers for elevation and azimuth drives would result in system instability.

For flexible antennas a quasiseparation of the flexible and tracking motions is observed. This property simplifies the controller design procedure. A controller for the flexible part is designed first, followed by a controller for the tracking part, with final corrections of the controller for the flexible part. The design of the controller for the flexible part is of sequential nature as well: a controller for each mode is designed separately.

The tracking performance requirements are not explicitly included in the definition of the performance index. Hence an optimal controller in the sense of minimization of the performance index may not satisfy the tracking requirements. Note that the closed-loop system dynamics depend heavily on the choice of the weighting matrix, as illustrated with the DSS-13 antenna in Fig. 2. First, the weight 10 for the integral of the antenna position, the weight 1 for the position itself, and the weight 0 for the flexible modes have been chosen. The antenna performance, characterized in this case by its step response in Fig. 2a, shows excessive flexible motion. Next, the weights are the same as those in the previous case, but the weights of the flexible modes are now set equal to 0.001. The closed-loop antenna performance in Fig. 2b shows a significant deterioration of the antenna tracking capabilities. Therefore, proper adjustment of weights is an important task, and it is crucial to determine the effect of weights on system performance. A method for determining this weight effect on the control system of the DSS-13 antenna is presented in this paper.

Quasiseparation of the Flexible and Tracking Subsystems

In this section the quasiseparation properties of an open-loop model (called also a rate-loop model) of a generic deep space network antenna are studied, based on the DSS-13 antenna model.

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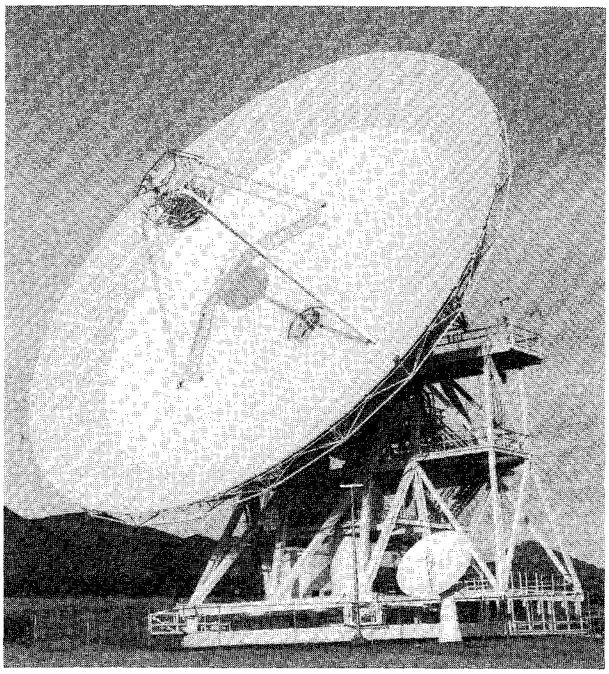

Fig. 1 DSS-13 antenna.

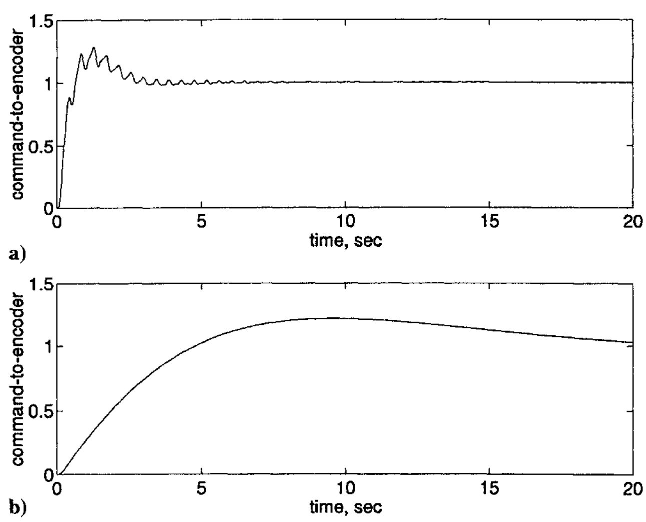

Fig. 2 Step responses of the DSS-13 antenna with LQ controller for a) small weights of flexible part and b) large weights of flexible part.

A balanced state-space representation (A, B, C) of the DSS-13 antenna is derived in Refs. 2 and 3. Its rate command input in elevation (u_{pe}) and azimuth (u_{pa}) is denoted $u_P^T = [u_{pe}\ u_{pa}]$, and the elevation (y_{pe}) and azimuth (y_{pa}) angles are its outputs, denoted $y_P^T = [y_{pe}\ y_{pa}]$. Its state vector $x = [x_t^T\ x_f^T]^T$ is divided into tracking x_t and flexible x_f states, where $x_t = [y_i^T, y_P^T]^T$, and $y_i = [y_{ie}, y_{ia}]^T$ is an integral of the elevation and azimuth position. The rate-loop representation (A, B, C) is divided, respectively, to the state vector x, thus

$$A = \begin{bmatrix} A_t & A_{tf} \\ 0 & A_t \end{bmatrix}, \quad B = \begin{bmatrix} B_t \\ B_f \end{bmatrix}, \quad C = [C_t \quad 0] \tag{1}$$

In this representation B_t is small in comparison with B_f (typically $\|B_t\| < 10^{-5}$ and $\|B_f\| > 1$). Also A_{tf} is small in comparison with A_t and A_f (typically $\|A_{tf}\| < 10^{-3}$, $\|A_f\| > 10$, and $\|A_t\| = 1$). Thus, the states of the tracking part are much weaker than the states of the flexible part. The strong states of the flexible subsystem and the weak states of the tracking subsystem are also visible in Fig. 3, which presents the transfer function plots of the rate-loop systems due to elevation rate command. This property is a foundation of the control design strategy described later.

In the LQ design the performance index J

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt \tag{2}$$

is minimized with respect to the controller gain matrix K. The minimum is obtained for $K = R^{-1}B^TS$, and S is a solution of the Riccati equation[6]

$$A^TS + SA - SBR^{-1}B^TS + Q = 0 \tag{3}$$

In the previous equations R is a positive definite input weight matrix, whereas Q is a positive semidefinite state weight matrix. It is assumed that $R = \rho I$, since both inputs (elevation and azimuth commands) are of equal importance, and the choice of $\rho = 1$ is made without loss of generality. Divide S and K into parts related to the triple (A, B, C) in Eq. (1),

$$S = \begin{bmatrix} S_t & S_{tf} \\ S_{tf}^T & S_f \end{bmatrix}, \quad K = [K_t \quad K_f] \tag{4}$$

so that Eq. (3) can be written as follows:

$$A_t^TS_t + S_tA_t - S_tB_tB_t^TS_t + Q_t - \Delta_{tf} = 0 \tag{5a}$$

$$A_t^TS_{tf} + S_{tf}A_f + S_tA_{tf} - K_t^TK_f = 0 \tag{5b}$$

$$A_f^TS_f + S_fA_f + S_fB_fB_f^TS_f + Q_f - \Delta_{ft} = 0 \tag{5c}$$

where

$$K_t = B_t^TS_t + B_f^TS_{tf}^T, \qquad K_f = B_t^TS_{tf} + B_f^TS_f \tag{6a}$$

$$\Delta_{tf} = S_tB_tB_f^TS_{tf}^T + S_{tf}B_fK_t$$
$$\Delta_{ft} = A_{tf}^TS_{tf} + S_{tf}^TA_{tf} - S_{tf}^TB_tK_f + S_fB_fB_t^TS_{tf} \tag{6b}$$

One can notice in Eqs. (6) that there exist weights Q_t and Q_f such that the gain K_f depends on the flexible subsystem only. Namely, for large enough Q_f, such that $\|Q_f\| \gg \|\Delta_{ft}\|$, the solution S_f of Eq. (5c) is independent of the tracking system, and for small Q_t one obtains $\|B_t^TS_{tf}\| \ll \|B_f^TS_f\|$. In terms of Eq. (6a) the latter inequality means that the gain K_f depends only on the flexible subsystem. However, the situation is not quite symmetric: there are no such Q_t and Q_f that the gain K_t depends only on the tracking subsystem. This follows from the fact that small Q_f and small Q_t are of different order, namely, $Q_f < 10^{-7}$ is

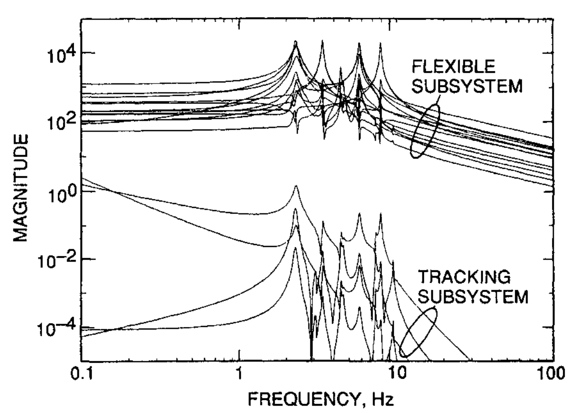

Fig. 3 Magnitude of transfer function of tracking and flexible subsystems for the elevation rate input.

considered small, whereas $Q_t < 1$ is considered small. Therefore, increasing Q_t to obtain $\|Q_t\| \gg \|\Delta_{tf}\|$, one obtains $\|B_f^T S_{ft}\|$ and $\|B_t^T S_t\|$ of the same magnitude. According to Eq. (6a) the latter fact means that the gain K_t depends on the flexible as well as on the tracking subsystem. This property can be validated by observation of the closed-loop transfer functions for different weights, as in Fig. 4, where the magnitude of the closed-loop transfer function (elevation angle-to-elevation command) for $Q_t = I_4$ and $Q_f = 0 \times I_{26}$ is shown as a solid line, for $Q_t = I_4$ and $Q_f = 10^{-7} \times I_{26}$ is shown as a dashed line, and for $Q_t = 10 \times I_4$ and $Q_f = 0 \times I_{36}$ is shown as a dot-dashed line. It follows from the plots that the variations of Q_f changed the properties of the flexible subsystem only, whereas the variations of Q_t changed the properties of both subsystems.

The independence of flexible system gains from tracking system properties is a consequence of small B_t, A_{tf}, and Q_t. However, the weight Q_t should be large enough to achieve the tracking performance requirements, and the increase of Q_t, causes the increased dependency of the gains on the tracking subsystem. In this case the previous independence is a quasi-independence, resulting in a quasiseparation of controller design. The design consists of the initial choice of relatively small weights for the tracking subsystem and determination of the controller gains of the flexible subsystem, followed by adjustment of weights of the tracking subsystem and a final tuning of the flexible weights.

Properties of an LQ Controller for Flexible Structures

In this application a linear system with distinct complex conjugate pairs of poles, and with small real parts of the poles, is considered a flexible structure. In the following a balanced state-space representation of a flexible structure is discussed. The balanced representation of flexible structures is close (but not identical) to a modal one.[7-9] For LQ synthesis purposes a balanced rather than modal representation is chosen, since the balanced reduction (necessary in controller design) yields more accurate results than the modal reduction, especially for closely spaced poles.[10]

Since the LQ controller for the flexible subsystem is determined separately from the tracking subsystem, in this section the flexible subsystem is considered only. Its state-space representation (A, B, C) is controllable and observable (subscript f is dropped in this section for simplicity of notation), and its controllability W_c and observability W_o grammians are equal

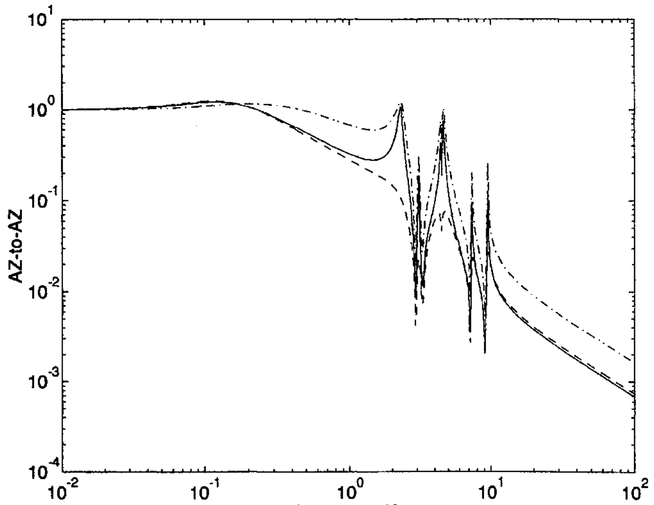

Fig. 4 Magnitude of transfer function of the closed-loop system for different weights: $Q_t = I$ and $Q_t = 0 \times I_{26}$ (solid line), $Q_t = I_4$ and $Q_f = 10^{-7} \times I_{26}$ (dashed line), and $Q_t = 10 \times I_4$ and $Q_f = 0 \times I_{26}$ (dot-dashed line).

and diagonal, $W_c = W_o = \Gamma^2$, where Γ is a positive definite diagonal matrix that satisfies the following Lyapunov equations:

$$A\Gamma^2 + \Gamma^2 A^T + BB^T = 0, \qquad A^T\Gamma^2 + \Gamma^2 A + C^T C = 0 \quad (7)$$

For a balanced flexible system with n components (or $2n$ states), the balanced grammian is diagonally dominant, $\Gamma \cong \mathrm{diag}\,(\gamma_1, \gamma_1, \gamma_2, \gamma_2, \ldots, \gamma_n, \gamma_n)$, and the matrix A is almost block diagonal,[7,8] with dominant 2×2 blocks on the main diagonal,

$$A \cong \mathrm{diag}(A_i), \quad A_i = \begin{bmatrix} -\zeta_i\omega_i & -\omega_i \\ \omega_i & -\zeta_i\omega_i \end{bmatrix}, \quad i = 1, \ldots, n \quad (8)$$

where ω_i is the ith natural frequency of the structure, and ζ_i is the ith modal damping. Combination of Eqs. (7) and (8) gives

$$2\gamma_i^2(A_i + A_i^T) \cong -B_i B_i^T \cong -C_i^T C_i \quad (9)$$

For the LQ controller it is assumed a diagonal weight matrix

$$Q = \mathrm{diag}(q_i I_2), \text{ with } 0 < q_i \ll 1, \quad i = 1, \ldots, n \quad (10)$$

In this case $S \cong \mathrm{diag}\,(s_i I_2)$ is the solution of Eq. (3), where

$$s_i = -0.5\gamma_i^{-2}(1 - \beta_i)$$
$$\beta_i = \sqrt{1 + 2q_i\gamma_i^2/\zeta_i\omega_i} \quad (11)$$
$$i = 1, \ldots, n$$

Proof is presented in the Appendix.

Next it will be shown that the weighting Q as in Eq. (10) shifts the ith pair of complex poles of the flexible structure and leaves the remaining pairs of poles almost unchanged. Only the real part of the pair of poles is changed (moving the pole apart from the imaginary axis, thus stabilizing the system), and the imaginary part of the poles remains unchanged. For the weight Q as in Eq. (10) the closed-loop pair of flexible poles $(\lambda_{cri}, \pm j\lambda_{cii})$ is obtained from the open-loop poles $(\lambda_{ori}, \pm j\lambda_{oii})$

$$(\lambda_{cri}, \pm j\lambda_{cii}) \cong (\beta_i\lambda_{ori}, \pm j\lambda_{oii}), \qquad i = 1, \ldots, n \quad (12)$$

where β_i is defined in Eq. (11). For proof see the Appendix.

The plots of β_i with respect to $\zeta_i\omega_i$ and γ_i are shown in Fig. 5. They show relatively large β_i even for small q_i, i.e., significant pole shift to the left. Also, since β_i increases with γ_i and decreases with $\zeta_i\omega_i$, there is a significant pole shift for highly observable and controllable modes with small damping. In terms of the transfer function profile, the weight q_i suppresses the resonant peak at frequency ω_i while leaving the natural frequency unchanged (see Fig. 6). Because of weak coupling between modes, the assignment of one mode insignificantly influences other modes, and thus the weight assignment can be done for each mode separately.

The following controller design algorithm is proposed

1) Determine the plant state-space representation in the form of Eq. (1), consisting of flexible and tracking parts.

2) Choose ad hoc but reasonably small weights for the tracking part $Q_t = Q_{tah}$.

3) For each balanced coordinate of the flexible part choose the weight q_i, $(i = 1, \ldots, n_f)$, and define the weight matrix $Q_{fi} = \mathrm{diag}\,(0, 0, \ldots, q_i, q_i, 0, \ldots, 0)$, so that the closed-loop system performance for the weight $Q_i = \mathrm{diag}\,(Q_{tah}, Q_{fi})$ is improved. For example, determine the weights q_i to impose the required pole shift or to suppress the ith resonant peak to the required level. Note that for small q_i only ith pair of poles is shifted (to the left), and the remaining poles are almost unaffected. Disregard the modes for which the weighting does not improve the closed-loop system performance.

4) For the already determined weight Q_f, tune weight Q_t to obtain improvements in tracking properties of the antenna (the

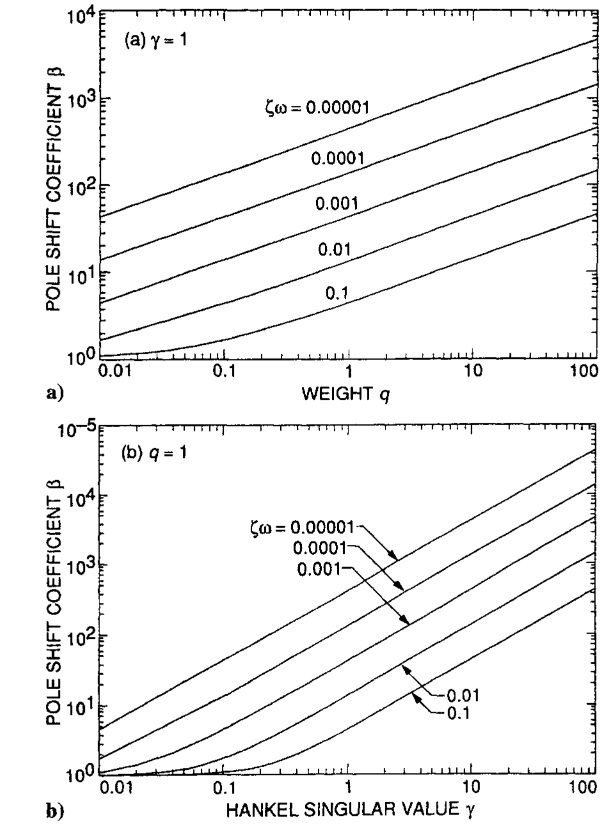

Fig. 5 Pole shift coefficient β vs a) weight q and Hankel singular value γ^2 and b) weight q and modal damping $\zeta\omega$.

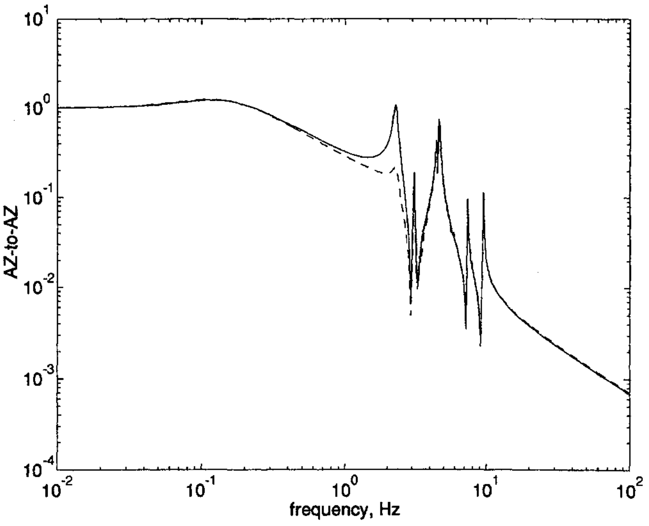

Fig. 6 Vibration suppression with a single weight.

magnitude of the transfer function equal to 1 for the required frequency bandwidth).

5) Adjust weights of the flexible subsystem, if necessary.

Controller Design for the DSS-13 Antenna

The recently designed DSS-13 34-m-dish antenna (Fig. 1) has significant flexible deformations observed during tracking operations. Its model[2,3] consists of 2 tracking states (azimuth and elevation angle) and 13 flexible modes (or 26 balanced states). A block diagram of its LQ control system is shown in Fig. 7.

The antenna tracking properties are shaped with the LQ controller. The properties are reflected in its step responses and transfer functions (input: elevation or azimuth command, output: elevation or azimuth encoder). In the step response the tracking performance is evaluated with the settling time, overshoot, and the presence of oscillatory motion; in the transfer function with bandwidth (the lowest frequency where the transfer function magnitude departs from 1 by 5%), roll-off rate outside the bandwidth, and the presence of resonance peaks. The "visual" evaluation of damping level—the presence of oscillatory motion in step response or resonance peaks in transfer fuction—could be replaced with numerical values of damping ζ_i, in Eq. (8), which should be close to the critical value.

The preliminary weights $q_{ie} = q_{pe} = q_{ia} = q_{pa} = 1$ for the tracking subsystem (for y_i and y_p) and zero weights for the flexible subsystem ($q_1 = q_2 = \ldots = q_{13} = 0$ for all 13 modes) have been chosen for the LQ controller design. The magnitudes of the closed-loop transfer function in Fig. 8 show excessive flexible motion of the antenna, which can be damped by adjusting weights for the flexible subsystem. Thus, for the tracking weights as before, the weight for the first mode (2.32 Hz) is chosen to be $q_1 = 10^{-7}$, and the remaining weights are zero, obtaining the closed-loop system responses as in Fig. 9. One can see that the 2.32-Hz resonance peak in the azimuth command response has disappeared. The elevation motion is unaffected, however, since this azimuth component is almost nonexistent in the elevation motion.

The weight should be chosen carefully. Too small weight (e.g., 3×10^{-9} in the case considered) will not suppress the resonant peak, as in Fig. 10a. Too large weight (e.g., 10^{-5}) will deteriorate the tracking performance: for the overweighted mode the transfer function is pressed down within wide fre-

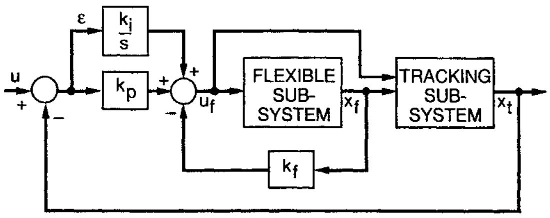

Fig. 7 Block diagram of the antenna closed-loop system.

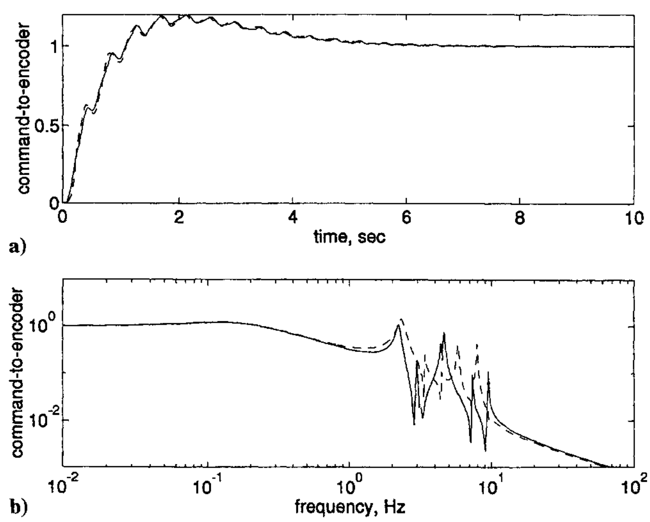

Fig. 8 a) Closed-loop step responses and b) magnitudes of transfer function, for unit proportional and integral weights in azimuth and elevation, and zero weights for flexible subsystem: ____, azimuth command input and azimuth encoder output; ----, elevation command input and elevation encoder output.

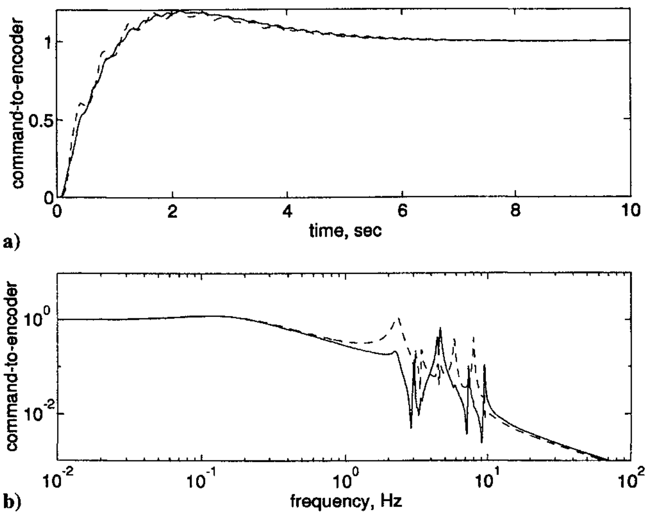

Fig. 9 a) Closed-loop step responses and b) magnitudes of transfer function, for weights the same as in Fig. 8 but $q_{f1} = 10^{-7}$: ____, azimuth command input and azimuth encoder output; ----, elevation command input and elevation encoder output.

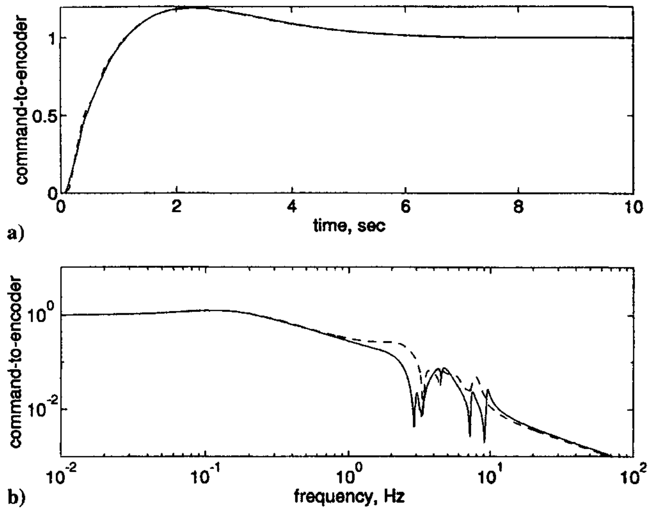

Fig. 11 a) Closed-loop step responses and b) magnitudes of transfer function, for weights the same as in Fig. 8 but $q_{f1} = \ldots = q_{f10} = 10^{-7}$: ____, azimuth command input, and azimuth encoder output; ----, elevation command input and elevation encoder output.

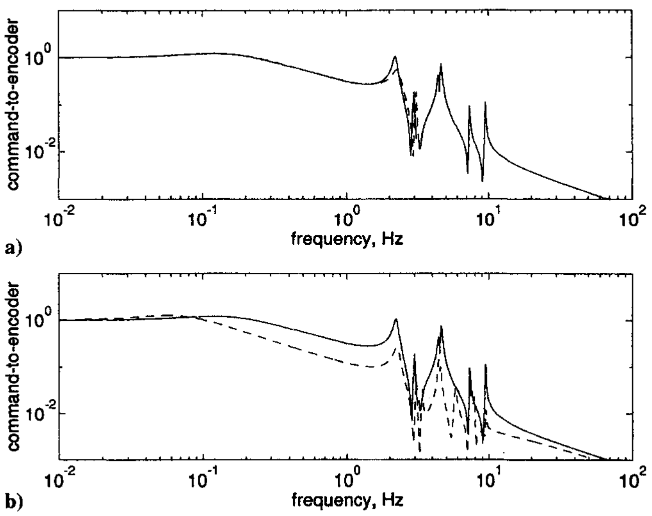

Fig. 10 First component: a) underweighted and b) overweighted.

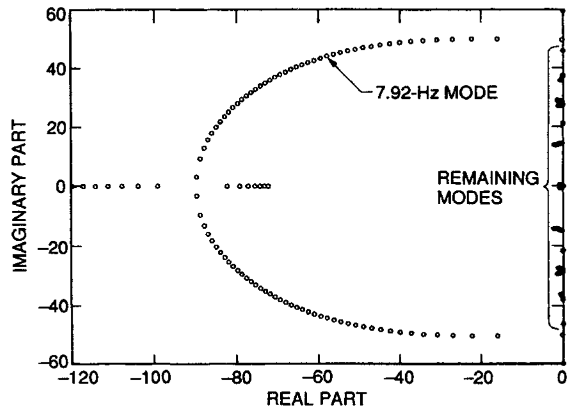

Fig. 12 Root locus for 7.92-Hz mode vs weight.

quency range (Fig. 10b). The proper weight suppresses the resonant peak, leaving the other peaks unchanged (Fig. 9).

Similar procedures have been applied for the second (2.64-Hz), third (4.26-Hz), fourth (3.77-Hz), fifth (7.88-Hz), sixth (4.47-Hz), seventh (3.38-Hz), eighth (5.98-Hz), ninth (7.32-Hz), and tenth (9.48-Hz) modes, with weight 10^{-7} for each mode. As a result, the suppression of the remaining flexible motion and resonant peaks is observed in Fig. 11. Weights for the remaining modes (11th–13th) have been set to zero.

The root locus of the closed-loop system due to weight variations of the 7.92-Hz mode is shown in Fig. 12. The figure shows the horizontal departure of poles into the left-hand side direction (stabilizing property). It confirms the properties of the weighted LQ design described previously in Eq. (12).

In the next step, the tracking properties of the system are improved by proper weight setting of the tracking subsystem. Namely, setting the integral weight to $q_{ie} = q_{ia} = 70$ and the proportional weight to $q_{pe} = q_{pa} = 100$ improves the system tracking properties, as shown in Fig. 13 (extended bandwidth, up to 2 Hz). However, by improving the tracking properties, the transfer function has been raised dramatically in the frequency region 1–3 Hz, which forces the first two modes located in this region to appear again in the step response. By sacrificing a bit of the tracking properties, the flexible motion in the step

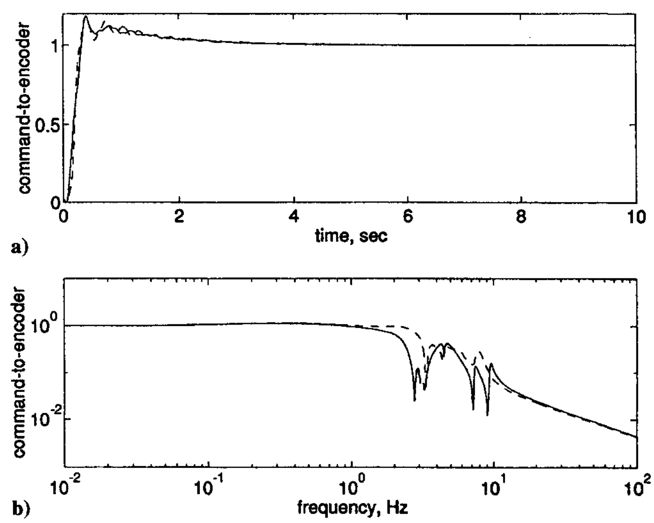

Fig. 13 a) Closed-loop step responses and b) magnitudes of transfer function, for proportional weight 100 and integral weight 70 (both in azimuth and elevation), and weights for flexible subsystem equal to 10^{-7}: ____, azimuth command input and azimuth encoder output; ----, elevation command input and elevation encoder output.

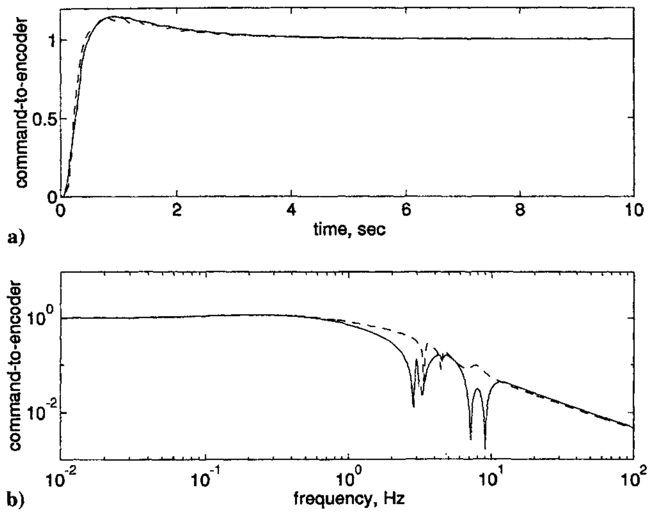

Fig. 14 a) Closed-loop step responses and b) magnitudes of transfer function, for proportional weight 100 and integral weight 70 (in azimuth and elevation), and for flexible weights $q_{f1} = \ldots = q_{f6} = 10^{-6}$, $q_{f7} = q_{f8} = 10^{-7}$, $q_{f9} = q_{f10} = 10^{-5}$: ____, azimuth command input and azimuth encoder output; ----, elevation command input and elevation encoder output.

response is removed. This is done by increasing the weights of the flexible subsystem, setting them as follows: $q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = 10^{-6}$, $q_7 = q_8 = 10^{-7}$, $q_9 = q_{10} = 10^{-5}$. The closed-loop system response with the satisfactory tracking performance is shown now in Fig. 14 (1-Hz bandwidth).

Conclusions

A new procedure for antenna controller design has been proposed. The antenna model is divided into flexible and tracking parts rather than into elevation and azimuth parts. In a sequential design strategy a controller for the flexible subsystem is designed first, followed by a controller design for the tracking subsystem, with a final tune-up of the flexible part. This approach results in a significant improvement of the performance of the antenna closed-loop system through a sequential weight adjustment of the state vector. The analytical relationships between weights and gains, and weights and root locus, have been derived in this paper. The order of the controller is determined through monitoring the closed-loop performance for each flexible mode. The design of a tracking controller for the recently built DSS-13 34-m-dish antenna illustrates the procedure.

Appendix

Proof of Eq. (11): For a flexible structure in balanced representation the state matrix A is diagonally dominant (with 2×2 blocks on the main diagonal), and for $R = I$ and Q as in Eq. (10), the solution S of the Riccati equation (3) is also diagonally dominant with 2×2 blocks S_i on the main diagonal, $S_i = s_i I_2$, $s_i > 0$, $i = 1, \ldots, n$. Thus, Eq. (3) turns into a set of the following equations, of which the solutions must be positive semidefinite:

$$s_i(A_i + A_i^T) - s_i^2 B_i B_i^T + q_i I_2 = 0, \quad i = 1, \ldots, n \qquad \text{(A1)}$$

For a balanced system $B_i B_i^T \cong -\gamma_i^2 (A_i + A_i^T)$, see Eq. (9), and for $A_i + A_i^T = -2\zeta_i \omega_i I_2$, see Eq. (8). Therefore Eq. (A1) is now

$$\gamma_i^2 s_i^2 + s_i - q_i/2\zeta_i\omega_i = 0, \qquad i = 1, \ldots, n \qquad \text{(A2)}$$

There are two solutions of Eq. (A2), but in our case

$$s_i = 0 \text{ is required for } q_i = 0 \qquad \text{(A3)}$$

Therefore Eq. (11) represents the unique solution of Eq. (A2) that satisfies Eq. (A3).

Proof of Eq. (12): For small q_i the matrix A of the closed-loop system is diagonally dominant $A_o = \text{diag}(A_{oi})$, $i = 1, \ldots, n$, and $A_{oi} = A_i - B_i B_i^T S_i$. Introducing Eq. (9) one obtains

$$A_{oi} = A_i + 2 s_i \gamma_i^2 (A_i + A_i^T) \qquad \text{(A4)}$$

and introducing A_i as in Eq. (8) into Eq. (A4), one obtains

$$A_{oi} = \begin{bmatrix} -\beta_i \zeta_i \omega_i & -\omega_i \\ \omega_i & -\beta_i \zeta_i \omega_i \end{bmatrix} \qquad \text{(A5)}$$

with β_i as in Eq. (11), which proves Eq. (12).

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